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Viscous and Joule heating effects on forced convection flow from radiate isothermal porous surfaces

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Abstract

Purpose – To highlight the effect of viscous and Joule heating on different ionized gases in the presence of magneto and thermal radiation effects.

Design/methodology/approach – The conservation equations are written for the MHD forced convection in the presence of thermal radiation. The governing equations are transformed into non-similar form using a set of dimensionless variables and then solved numerically using Keller box method.

Findings – The increasing of fluid suction parameter enhances local Nusselt numbers, while the increasing of injection parameter decreases local Nusselt numbers. The inclusion of thermal radiation increases the heat transfer rate for both ionized gases suction or injection. The presence of magnetic field decreases the heat transfer rate for the suction case and increases it for the injection case. Finally, the heat transfer rate is decreased due to viscous dissipation.

Research limitations/implications – The combined effects of both viscous and Joule heating on the forced convection heat transfer of ionized gases for constant surface heat flux surfaces can be investigated.

Practical implications – A very useful source of coefficient of heat transfer values for engineers planning to transfer heat by using ionized gases.

Originality/value – The viscous and Joule heating of ionized gases on forced convection heat transfer in the presence of magneto and thermal radiation effects are investigated and can be used by different engineers working on industry.

Keywords Gases, Convection, Heat transfer

Paper type Research paper

Nomenclature

a = Stefan-Boltzmann constant
B_0 = magnetic field flux density, Wb/m^2
Cf_x = local skin friction factor
c_p = specific heat capacity
f = dimensionless stream function
Ec = Eckert number, $u_\infty^2/c_p(T_w - T_\infty)$
Ec$'$ = modified Eckert number, EcPr
g = gravitational acceleration
h = heat transfer coefficient
k = thermal conductivity
L = length of the plate
Ha_x^2 = magnetic influence number, $\sigma B_0^2 x/\rho \upsilon$
Nu$_x$ = local Nusselt number, hx/k
$\overline{\text{Nu}}$ = average Nusselt numbers, $\bar{h}L/k$
p = pressure
Pr = Prandtl number
$q_w(x)$ = local surface heat flux

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Re_x	= local Reynolds number, $u_\infty x/\upsilon$	α_R	= Rosseland mean absorption coefficient
Pe_x	= local Peclet number, $u_\infty x/\alpha$	β	= coefficient of thermal expansion, $-1/\rho(\partial\rho/\partial T)_\mathrm{p}$
R_d	= conduction-radiation parameter, $k\alpha_\mathrm{R}/4aT_\infty^3$	χ	= non-similarity parameter, $(-2v_\mathrm{w}/u_\infty)\mathrm{Pe}_x^{1/2}$
T	= temperature	η	= pseudo-similarity variable
T_∞	= free stream temperature	Θ	= dimensionless temperature
T_w	= wall temperature	Θ_w	= ratio of surface temperature to the ambient temperature, T_w/T_∞
u,v	= velocity components in x-and y-directions	υ	= kinematic viscosity
u_∞	= free stream velocity	ρ	= fluid density
v_w	= porous wall suction or injection velocity	σ	= electrical conductivity
x,y	= axial and normal coordinates	τ_w	= local wall shear stress
Greek symbols		ψ	= dimensional stream function
α	= thermal diffusivity		

1. Introduction

An ionized gas, whether it occurs as a result of an elevation of temperature or by suitable seeding process, is electrically conductive radiate and influenced by different magnetic fields. The study of magnetohydrodynamic viscous radiate flows has important industrial, technological and geothermal applications such as high-temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators, and power generation systems. The MHD-forced convection heat transfer problems have been studied by Sparrow and Cess (1961), Romig (1964), Garandet *et al.* (1992), and Takhar and Ram (1994). The effect of radiation on heat transfer problems has been studied by Soundalgekar *et al.* (1960), Hossain and Takhar (1996), Hossain *et al.* (1999), and Raptis (2001). A set of nonsimilar solutions for different mixed convection heat transfer boundary layers for different geometries embedded in saturated porous medium has been developed by Aldoss *et al.* (1993a, b, 1994), Hsich *et al.* (1993) and Duwairi *et al.* (1997). Recently, Raptis and Perdikis (2000) investigated the MHD free convection flow in the presence of thermal radiation, Duwairi and Damseh (2004a, b) studied both effects on natural convection and mixed convection heat transfer problems with fluid suction or injection from vertical surfaces, Duwairi (2004) studied the effect of both magneto forces and thermal radiation on forced convection heat transfer from constant surface heat flux surfaces in the absence of viscous and Joule heating effects.

In the present analysis, a non-similar solution for the viscous and Joule heating effects on forced convection flow of ionized gases is investigated. The governing equations are transformed using a set of dimensionless variables and then solved numerically using Keller box method.

2. Analysis

The analysis is carried out for the case of uniform surface temperature T_w which is placed in a fluid at temperature T_∞ with free stream velocity u_∞ and in a uniform magnetic field B_0 (independent of x). The x coordinate is measured from the leading edge of the plate, the y coordinate is measured normal to the plate and u and v are the corresponding velocities in the x and y directions, respectively, v_w is the suction or injection velocity. The flow is steady, laminar, incompressible and two-dimensional, the fluid is assumed to be gray, emitting and absorbing heat but not scattering. These conditions do not lead to a similar solution of the laminar boundary-layer equations. Therefore, solutions of the governing equations have been obtained utilizing a non-similar approach. Under boundary layer and Rosseland diffusion approximations, the extended continuity, momentum and energy equations by Holman (1999) are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\sigma B_0^2(u-u_\infty)=\mu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\rho c_{\mathrm{p}}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)-\sigma B_0^2u^2=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{16a}{3\alpha_{\mathrm{R}}}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The radiation heat transfer fluxes for an optically thick fluid are included in the energy equation as described by Ali *et al.* (1984). The boundary conditions can be written as

$$\begin{aligned} &y=0, \qquad u=0,\ \ v=\pm v_{\mathrm{w}},\ \ T=T_{\mathrm{w}} \\ &y\to\infty, \qquad u=u_\infty,\ \ T=T_\infty \end{aligned} \tag{4}$$

The minus sign for the conductive gray fluid vertical velocity means the suction from the porous wall, where the plus sign means the gray fluid injection. In the above system of equations and in order to satisfy the continuity equation define the stream function as $u=\partial\psi/\partial y$, and $v=-\partial\psi/\partial x$, the following dimensionless variables are also introduced in the transformation

$$\eta=(y/x)\mathrm{Pe}_x^{1/2}, \quad \chi=\chi(x) \tag{5}$$

$$\psi=\alpha\mathrm{Pe}_x^{1/2}f(\chi,\eta), \quad \Theta(\chi,\eta)=\frac{T-T_\infty}{T_{\mathrm{w}}-T_\infty} \tag{6}$$

Using the non-similar variables equations (5) and (6) in the governing equations ((1)-(4)), the momentum and energy equations are

$$f'''+(1/2)ff''+\frac{Ha_x^2}{\mathrm{Re}_x}(1-f')=(1/2)\chi\left(f'\frac{\partial f'}{\partial\chi}-f''\frac{\partial f}{\partial\chi}\right) \tag{7}$$

$$\begin{aligned} &\Theta''+\left[\left\{(4/3R_d)(1+(\Theta_{\mathrm{w}}-1)\Theta)^3\right\}\Theta'\right]'+(1/2)f\Theta'+\frac{Ha_x^2}{\mathrm{Re}_x}\mathrm{Ec}'(f')^2+\mathrm{Ec}'(f'')^2 \\ &=(1/2)\chi\left(f'\frac{\partial\Theta}{\partial\chi}-\Theta'\frac{\partial f}{\partial\chi}\right) \end{aligned} \tag{8}$$

where

$$\chi(x)=-(2v_{\mathrm{w}}/u_\infty)\mathrm{Pe}_x^{1/2}$$

the corresponding boundary conditions are

$$f'(\chi,0)=0,\ \ f(\chi,0)=\pm\chi,\ \ \Theta(\chi,0)=1$$
$$f'(\chi,\infty)=1,\ \ \Theta(\chi,\infty)=0 \tag{9}$$

The corresponding dimensionless groups appeared in the governing equations are $Ha_x^2=\sigma B_0^2 x/\rho\upsilon$, $\mathrm{Ec}=u_\infty^2/cp(T_\mathrm{w}-T_\infty)$, $\mathrm{Pr}=\upsilon/\alpha$, $\mathrm{Ec}'=\mathrm{EcPr}$, $\Theta_\mathrm{w}=T_\mathrm{w}/T_\infty$, and $R_d=k\alpha_\mathrm{R}/4aT_\infty^3$. The minus sign for the boundary condition means the injection where the plus sign means the gray fluid suction through the porous wall. The primes denote partial differentiations with respect to η. Note that the nonsimilar parameter $\chi(x)$ will reflect the effect of the gray fluid suction or injection from the porous plate, and the case $\chi=0$ means that the plate is impermeable. The conduction-radiation parameter R_d will reflect the effect of absent radiation from the porous wall and between the gray fluid layers on the MHD forced convection heat transfer problem under consideration, the radiation effect is absent when $R_d\rightarrow\infty$. Note that the Joule and viscous dissipation heating effects appeared in the energy equation as $Ha_x^2/\mathrm{Re}_x\mathrm{Ec}'$ and Ec'. The values of Ha_x^2/Re_x are assigned 0 to remove these effects. Some of the physical quantities of practical interest include the velocity component u and v in the x- and y-directions, the wall shear stress, $\tau_\mathrm{w}=\mu(\partial u/\partial y)_{y=0}$, and the surface heat flux $q_\mathrm{w}(x)=-\{k+(16a/3\alpha_\mathrm{R})T^3\}(\partial T/\partial y)_{y=0}=h(T_\mathrm{w}-T_\infty)$. They are given by

$$u=u_\infty f'(\chi,\eta) \tag{10}$$

$$v=-v_\mathrm{w}\chi^{-1}\left\{f(\chi,\eta)+2\chi\frac{\partial f}{\partial\chi}-\eta f'(\chi,\eta)\right\} \tag{11}$$

$$Cf_x\mathrm{Pr}^{-1}\mathrm{Pe}_x^{1/2}=2f''(\chi,0) \tag{12}$$

$$\frac{\mathrm{Nu}_x\mathrm{Pe}_x^{-1/2}}{\left(1+(4/3R_d)\Theta_\mathrm{w}^3\right)}=-\Theta'(\chi,0) \tag{13}$$

The average Nusselt numbers along the porous plate can be obtained from the local Nusselt number, equation (13). The end result is given by

$$\frac{\overline{\mathrm{Nu}}\mathrm{Pe}_L^{-1/2}}{\left(1+(4/3R_d)\Theta_\mathrm{w}^3\right)}=-2\chi_L^{-1}\int_0^{\chi_L}\Theta'(\chi,0)\,\mathrm{d}\chi \tag{14}$$

where χ_L and Pe_L are the χ_x and Pe_x calculated at $x=L$

3. Numerical solutions

The partial differential equations ((7) and (8)) under the boundary conditions (9) are solved numerically by using an implicit iterative tridiagonal finite-difference method (Cebeci and Bradshaw, 1984). In this method, any quantity g at point (χ_n,η_j) is written as g_j^n. Quantities and derivatives at the midpoints of grid segments are approximated to second order as

$$g_j^{n-1/2} = \frac{1}{2}\left(g_j^n + g_j^{n-1}\right), \quad g_{j-1/2}^n = \frac{1}{2}\left(g_j^n + g_{j-1}^n\right) \tag{15}$$

$$\left(\frac{\partial g}{\partial \chi}\right)_j^{n-1/2} = \Delta\chi^{-1}\left(g_j^n - g_j^{n-1}\right), \quad (g')_{j-1/2}^n = \Delta\eta^{-1}\left(g_j^n - g_{j-1}^n\right) \tag{16}$$

where g is any dependent variable and n and j are the node locations along the χ and η directions, respectively. First the third-order partial differential equation is converted in the first order by substitutions $f' = s$, and $s' = w$, the difference equations that are to approximate the previous equations are obtained by averaging about the midpoint $(\chi_n, \eta_{j-1/2})$, and those to approximate the resulting equations by averaging about $(\chi_{n-1/2}, \eta_{j-1/2})$. At each line of constant χ, a system of algebraic equations is obtained. With the nonlinear terms evaluated at the previous station, the algebraic equations are solved iteratively. The same process is repeated for the next value of χ and the problem is solved line by line until the desired χ value is reached. A convergence criterion based on the relative difference between the current and previous iterations is employed. When this difference reaches 10^{-5}, the solution is assumed to have converged and the iterative process is terminated. The effect of the grid size $\Delta\eta$ and $\Delta\chi$ and the edge of the boundary layer η_∞ on the solution had been examined. The results presented here are independent of the grid size and the η_∞ up to the fourth decimal point.

The accuracy of the selected method is tested by comparing the results with those of the classical forced-convection problem over a vertical isothermal permeable plate (Holman, 1999). Table I shows a comparison between the Nusselt numbers, it is seen that the present results are in a good agreement.

4. Results and discussion

In this paper, the viscous and Joule heating effects on forced convection flow of ionized gas adjacent to radiate porous wall are investigated. Figures 1 and 2 show the dimensionless velocity and temperature profiles inside the boundary layer for different suction parameter $\chi = 1, 3$ or injection parameter $\chi = -1, -3$ and $\Theta_w = 1.5$, $R_d = 1$, $Ec' = 0.005$, $Ha_x^2/\mathrm{Re}_x = 1$. The increasing of suction parameter decreases the boundary layer thickness and increases temperature gradients, so the heat transfer rates are enhanced. The increasing of the injection parameter increases the boundary layer thickness and decreases temperature gradients near the porous wall, so the heat transfer rates are decreased, note that the case of $\chi = 0$ corresponds to the impermeable radiate plate. The effects of the magnetic influence parameter $Ha_x^2/\mathrm{Re}_x = 0, 1, 3$

	$\mathrm{Nu}_x/\sqrt{\mathrm{Re}_x}$	
χ	Present method	Holman (18)
−0.1	0.23895	0.2400
−0.4	0.09878	0.0950
−0.6	0.01057	0.0100
0.0	0.33234	0.3320
1.0	0.27632	0.2850
4.0	1.00327	1.0000
5.0	3.62368	3.6000

Table I. Values of $\mathrm{Nu}_x/\sqrt{\mathrm{Re}_x}$ $\mathrm{Pr} = 0.71$, $R_d \to \infty$, $M = 0$ and $Ec' = 0$

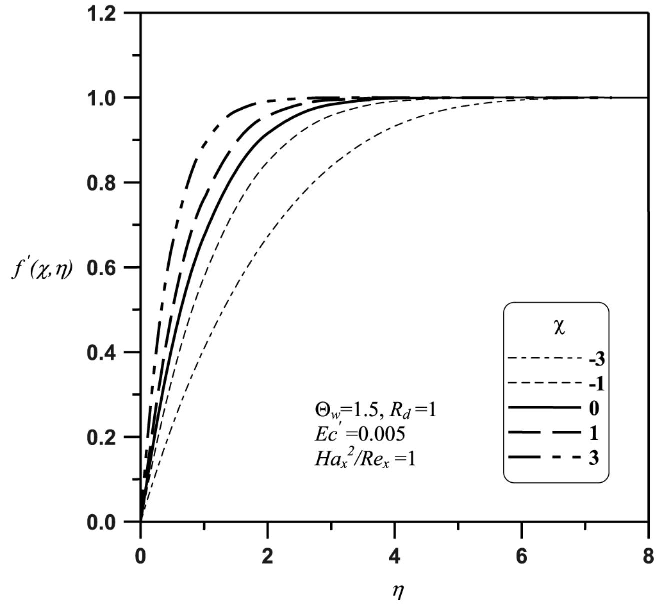

Figure 1.
Dimensionless velocity profiles for different χ

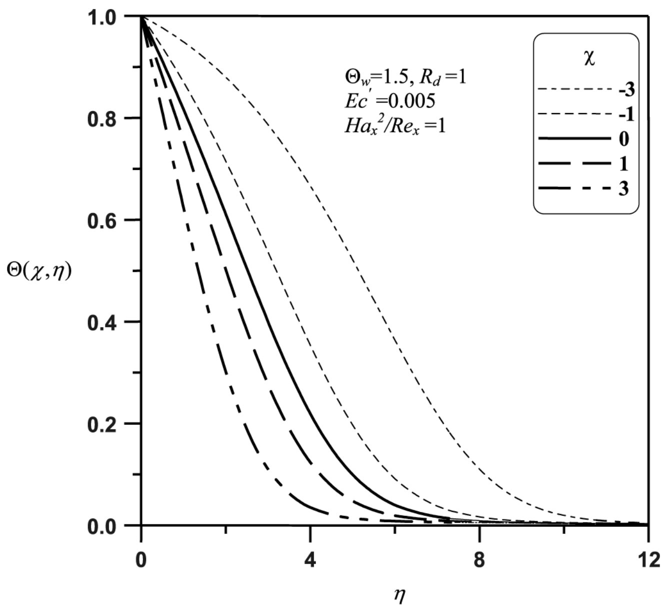

Figure 2.
Dimensionless temperature profiles for different χ

on both the dimensionless velocity and temperature profiles are drawn in Figures 3 and 4, respectively, for $\Theta_{\rm w} = 1.5$, $R_d = 1$, $\mathrm{Ec}' = 0.005$, $\chi = 1, -1$. The increasing of the magnetic influence parameter decreases the velocity inside boundary layer due to retarding effect of magnetic forces on both suction or injection porous plates, also the

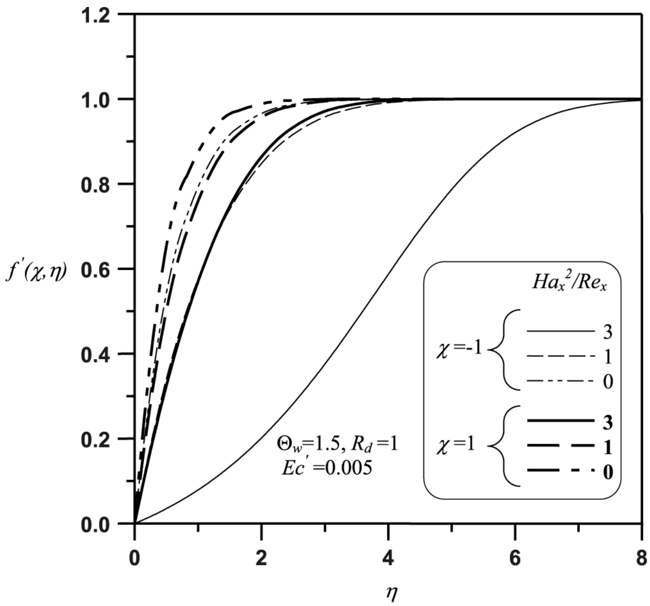

Figure 3.
Dimensionless velocity profiles for different Ha_x^2/Re_x

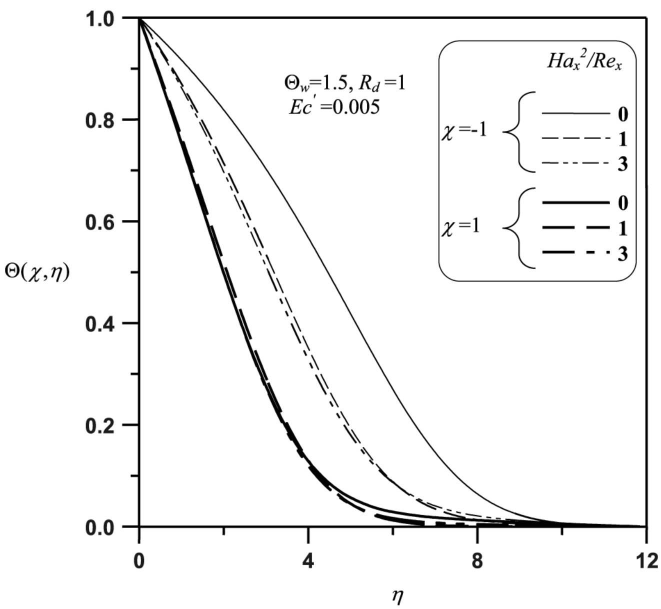

Figure 4.
Dimensionless temperature profiles for different Ha_x^2/Re_x

heat transfer rates are decreased for the suction porous plate case and increased for the injection porous plate case. The reason for this behavior is that the magnetic field had negligible effects in raising the fluid temperature for the injection porous plate, and the Joule heating effect can be neglected. The effects of the conduction-radiation parameter

$R_d = 1, 5, 50$ on the dimensionless temperature profiles are drawn in Figure 5 for $\Theta_w = 1.5$, $\chi = 1, -1$, $Ec' = 0.005$, $Ha_x^2/Re_x = 1$, it is found that the increasing of conduction-radiation parameter increases temperature gradients near the porous wall for both case of fluid suction or injection, which increases heat transfer rates, this is due to the fact that radiation effects increase temperatures of ionized gases and the absence of radiation defines small temperatures.

Figure 6 shows the effect of the viscous dissipation term $Ec' = 0.001, 0.005, 0.01$ included in the energy equation on the dimensionless temperature profiles for $\Theta_w = 1.5$, $\chi = 1, -1$, $R_d = 1$, $Ha_x^2/Re_x = 1$. The increasing of modified Eckert numbers broadens the temperature distribution inside the boundary layer and decreases heat transfer rates. The effect of the surface temperature parameter $\Theta_w = 1.1, 1.5, 2$ is shown in Figure 7 for $Ec' = 1.5$, $\chi = 1, -1$, $R_d = 1$, $Ha_x^2/Re_x = 1$; the increasing of this parameter heats the conductive gray fluid and broadens the temperature inside the boundary layer, however, the increasing of this parameter enhances heat transfer rate as it appears from equations (13) and (14).

Figures 8 and 9 show the variation of local Nusselt numbers for different conduction-radiation parameter and different magnetic influence parameter. Figure 8 shows that the increasing of suction parameter enhances local Nusselt numbers, while the increasing of injection parameter decreases local Nusselt numbers. The effect of absent radiation from the porous wall is to decrease local Nusselt numbers, due to important role played by radiation in transferring heat. Figure 9 shows that the increasing of magnetic influence parameter increases local Nusselt numbers for the injection case and decreases it for the suction case, this is due to negligible Joule heating effect and important retardation of magneto forces during fluid injection.

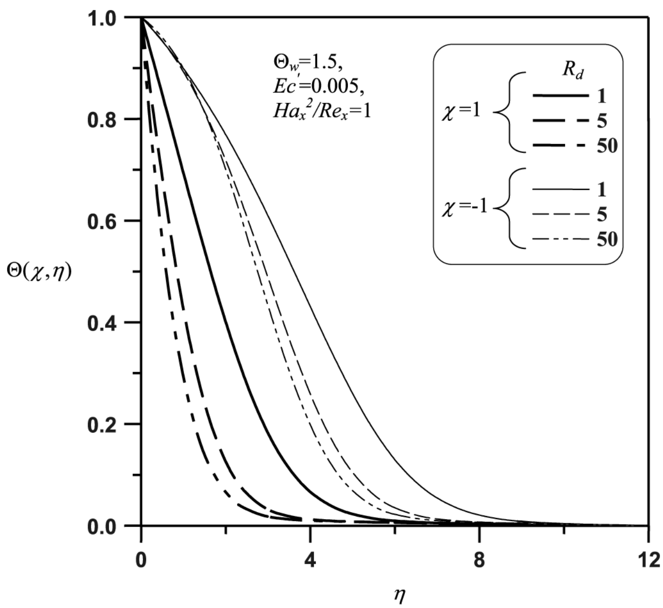

Figure 5.
Dimensionless temperature profiles for different R_d

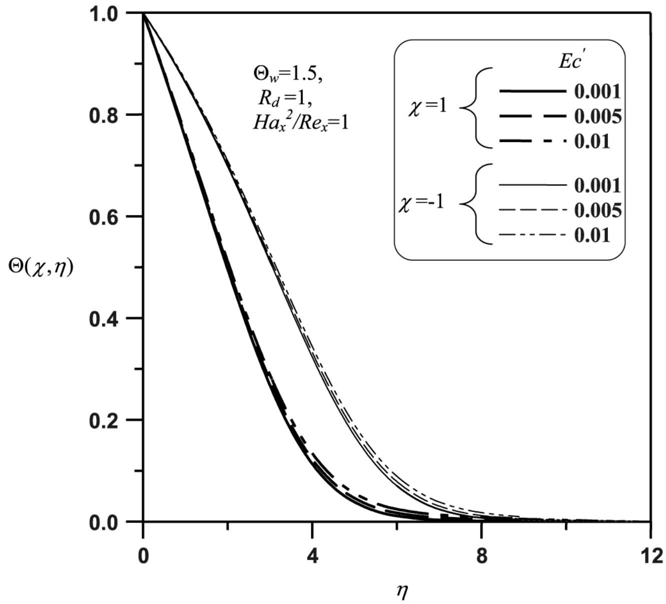

Figure 6.
Dimensionless temperature profiles for different Ec′

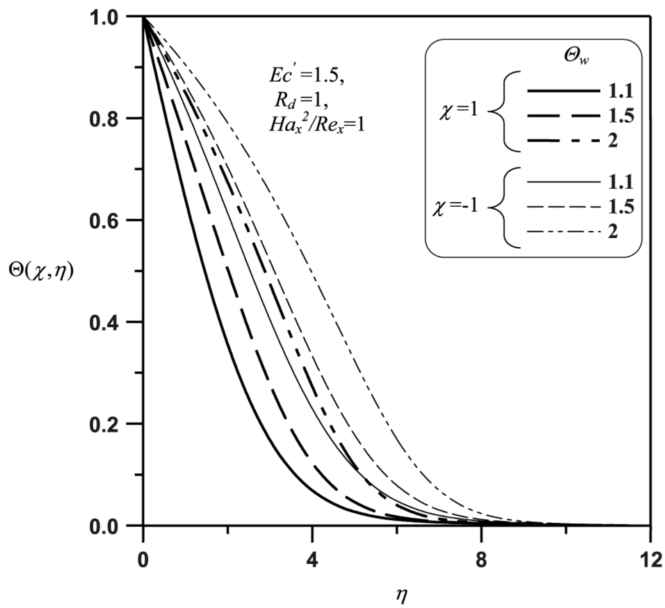

Figure 7.
Dimensionless temperature profiles for different Θ_w

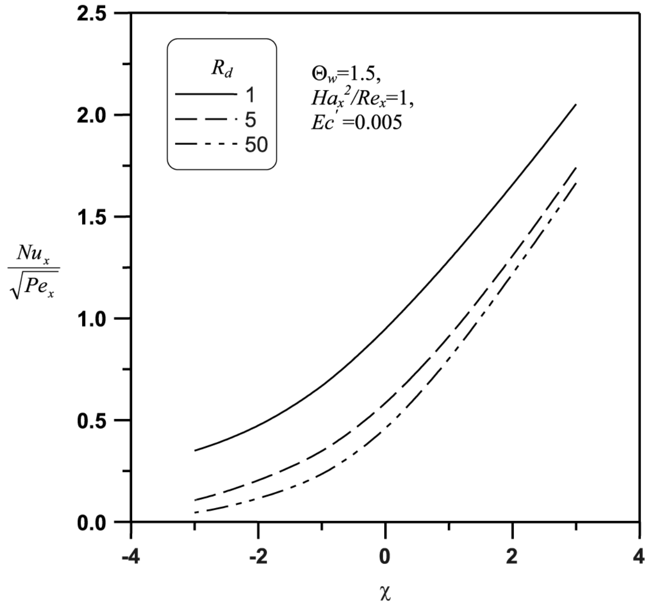

Figure 8.
Local Nusselt number variations for different R_d

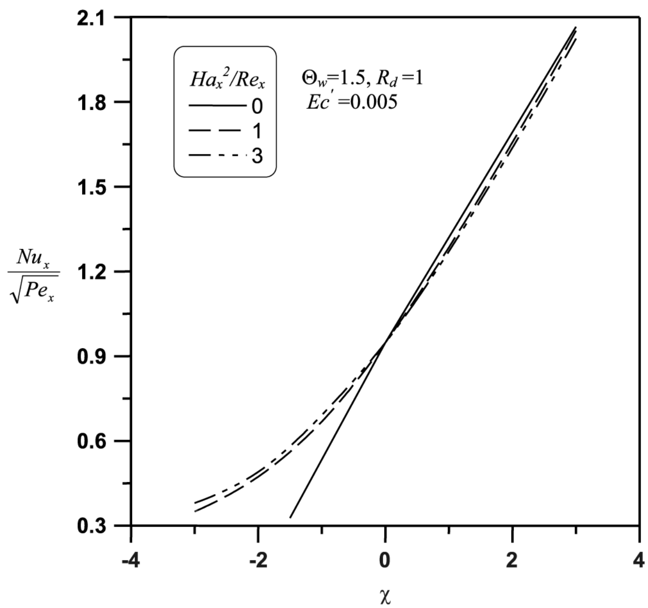

Figure 9.
Local Nusselt number variations for different Ha_x^2/Re_x

Also it is due to the important Joule heating effect and negligible retardation of magneto forces during fluid suction.

5. Conclusions

In this paper numerical solutions are presented for viscous and Joule heating effects on forced convection flow of ionized gases adjacent to isothermal porous surfaces. The partial differential equations are transformed into non-similar boundary layer equations which are solved by Keller box method, and it is found that:

(1) The increasing of fluid suction parameter enhances local Nusselt numbers, while the increasing of injection parameter decreases local Nusselt numbers; this is due to small and large thermal boundary layer thicknesses, respectively.

(2) The presence of radiation serves to introduce two extra parameters into the problem, the R_d and Θ_w. An increase in R_d serves to decrease the heat transfer rate, due to negligible an important role played by radiation in transferring heat between ionized gas layers, while the increase in Θ_w serves to decrease temperature gradient near porous wall and increases local Nusselt numbers.

(3) The presence of magnetic field decreases the heat transfer rate for the suction case and increases it for the injection case; this is due to negligible Joule heating effect for the injection porous plate.

(4) Finally, the heat transfer rate is decreased due to viscous dissipation effect in heating ionized gases for both cases of fluid suction or injection.

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